Name:_____ Date:_____

SOLUTION Math 12P: HW Section 1.3 Horizontal and Vertical Reflections

1. When you reflect a function horizontally, which axis is it reflected over? The X axis or Y axis? Explain

A horizontal reflection would reflect a function over the Y-axis. When we reflect horizontally, the graph is transferred from left to right or vice versa. This would result in a reflection over the Y-Axis.

2. When you reflect a function vertically, which axis is it reflected over? The X axis or Y axis? Explain

A vertical reflection would reflect a function over the X-axis. Likewise, a vertical reflection would reflection the graph up/down. Ie: A function above the Xaxis would be reflected below the X-axis.

3. When a function is reflected horizontally, which coordinate is affected? The X or Y coordinate? Explain

A horizontal reflection over the Y-axis would only affect the X-coordinates. A positive X-coordinate would become negative, and a negative X-coordinate would become positive. The Y-coordinates would NOT be affected at all.

4. When a function is reflected vertically, which coordinate is affected? The X or Y coordinate? Explain

A vertical reflection over the X-axis would only affect the Y-coordinates. Positive Y coordinate becomes negative and vice versa. The X-coordinates would NOT be affected in a Vertical reflection.

5. Indicate whether if the following is a vertical, horizontal reflection, both, or either. Explain:

i) $y = x^2 \rightarrow y = -x^2$

Vertical Reflection b/c $y \rightarrow -y$

ii) $y = \sqrt{x} \rightarrow y = \sqrt{-x}$

Horizontal Reflection b/c $x \rightarrow -x$

iii) $y = mx + b \rightarrow y = -mx - b$

Vertical Reflection b/c $y \rightarrow -y$

iv) $y = \frac{1}{x} \rightarrow y = -\frac{1}{x}$

THis can be either:
Horizontal Reflection b/c $x \rightarrow -x$
Vertical Reflection b/c $y \rightarrow -y$

v) $y = x^3 + 2 \rightarrow y = -x^3 + 2$

Horizontal Reflection b/c $x \rightarrow -x$

$y = x^3 + 2 \rightarrow y = (-x)^3 + 2$

$\rightarrow y = -(x)^3 + 2$

vi) $y = (x-3)^2 + 4 \rightarrow y = -(x-3)^2 - 4$

Vertical Reflection b/c $y \rightarrow -y$

vii) $y = (x-2)^2 - 5 \rightarrow y = (x+2)^2 - 5$

Horizontal Reflection b/c $x \rightarrow -x$

$y = (x-2)^2 - 5 \rightarrow y = (-x-2)^2 - 5$

$\rightarrow y = (-(x-2))^2 - 5$

$\rightarrow y = (-1)^2 (x-2)^2 - 5$

$\rightarrow y = (x-2)^2 - 5$

viii) $y = |x+2| \rightarrow y = -|x-2|$

Both reflections: Horizontal and Vertical

$y = |x+2| \rightarrow -y = |-x+2|$

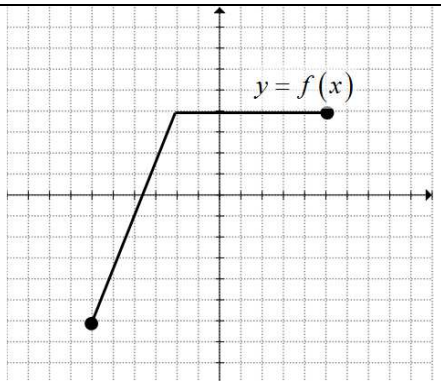
$\rightarrow -y = |-(x-2)|$

$\rightarrow -y = |-1||(x-2)|$

$\rightarrow -y = |(x-2)|$

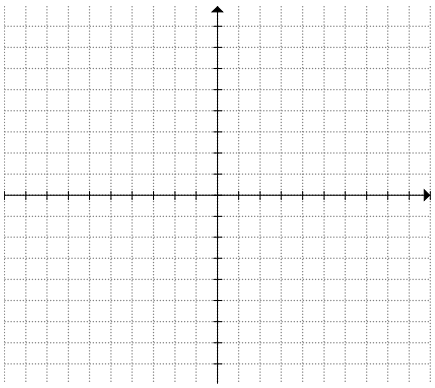
$\rightarrow y = -|(x-2)|$

Given the graph of $y = f(x)$, draw the resulting image after each transformation:



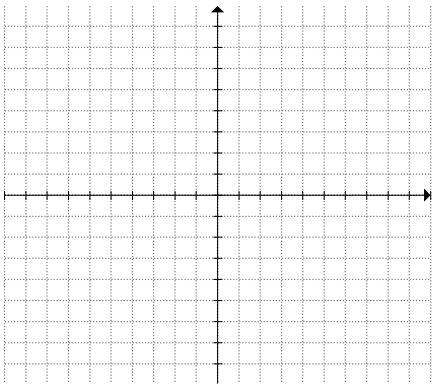
The KEY points that should be are (-6, -6) , (-2, 4) and (5,4)

i) $y = -f(x)$

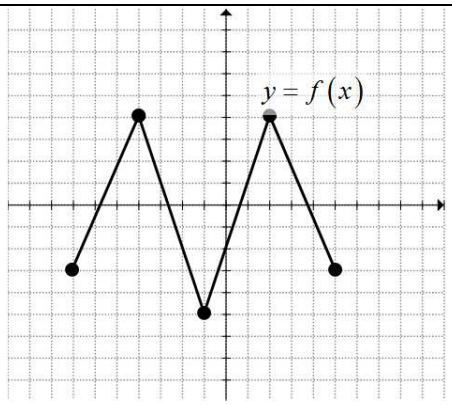


i) This is a vertical reflection, the Y-coordinates will all be multiplied by -1. (a,b) → (a,-b)

ii) $y = f(-x)$

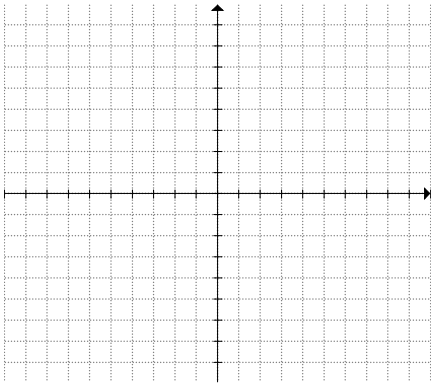


ii) This is a horizontal reflection, the X-coordinates will all be multiplied by -1. (a,b) → (-a, b)



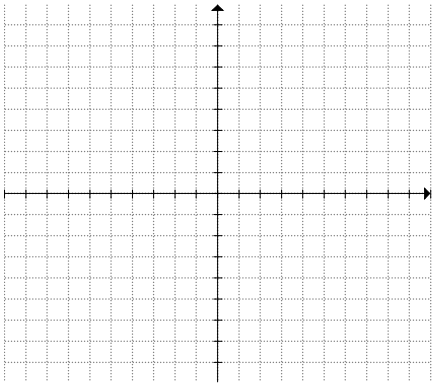
The KEY points that should be are (-7, -3) , (-4, 4) , (-1, -5) , (2, 4) and (5,-3)

i) $y = -f(x) + 3$

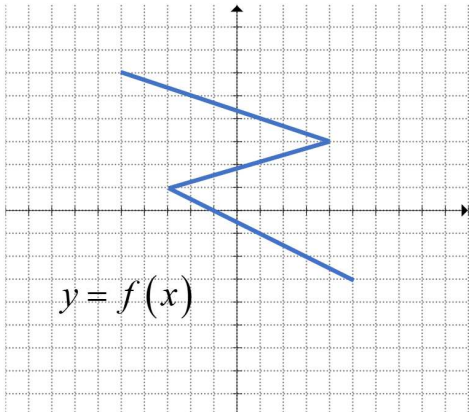


there is a 1) Vertical relfection, Y-coordinates are multiplied by -1. 2) Vertical shift of 3 units up that is performed after the reflection, all the Y-coordinates will increase by 3

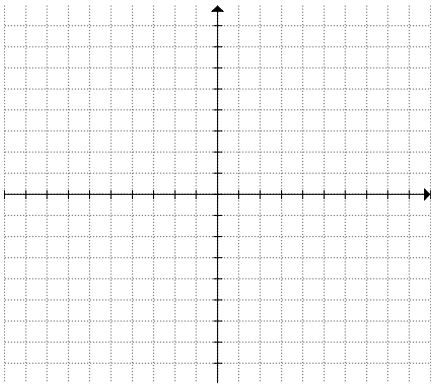
ii) $y = f(-x)$



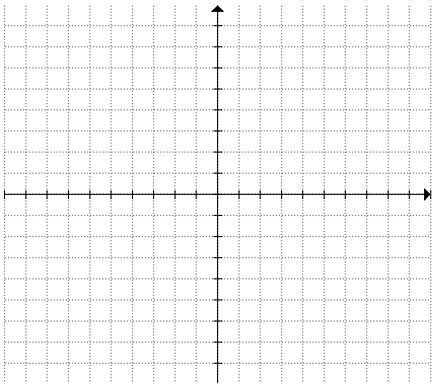
ii) This is a horizontal reflection, the X-coordinates will all be multiplied by -1. (a,b) → (-a, b)



i) $y = -f(x)$



ii) $y = -f(-x)$



6. The following transformations from the function on the left to the function on the right involve translations and reflections. List the transformations in order:

a) $y = |x| \rightarrow y = -|x-2|$

1. Vertical reflection $y \rightarrow -y$ $y = |x| \rightarrow y = -|x|$

2. Horizontal shift 2 units right $x \rightarrow x-2 \rightarrow y = -|x-2|$

b) $y = \sqrt{x} \rightarrow y = \sqrt{3-x} - 7$

For this question, arrange the terms inside the radical so that the "x" is on the left and the 3 on the right:

$y = \sqrt{x} \rightarrow y = \sqrt{-x+3} - 7$

Now factor the negative sign so that it is easier to read the transformations

$y = \sqrt{x} \rightarrow y = \sqrt{-(x-3)} - 7$

1. Horizontal reflection $x \rightarrow -x$ $y = \sqrt{x} \rightarrow y = \sqrt{-x}$

2. Horizontal shift of 3 units right $x \rightarrow x-3 \rightarrow y = \sqrt{-(x-3)}$

3. Vertical shift of 7 units down $y \rightarrow y+7$

$\rightarrow y+7 = \sqrt{-(x-3)}$

$\rightarrow y = \sqrt{-(x-3)} - 7$

c) $y = 3x + 2 \rightarrow y = -3x - 2$

If you factor out the negative sign from both terms, it makes it easier to recognize that it is a vertical reflection:

$y = -3x - 2 \rightarrow y = -(3x + 2)$

d) $y = x^2 \rightarrow y = -x^2 - 2x - 4$

YOU will need to Complete the Square for this question:

$y = -x^2 - 2x - 4 \rightarrow y = -(x + 2x) - 4$

$\rightarrow y = -(x + 2x + 1 - 1) - 4$

$\rightarrow y = -(x + 2x + 1) + 1 - 4$

$\rightarrow y = -(x + 1)^2 - 3$

So our question now becomes: $y = x^2 \rightarrow y = -(x + 1)^2 - 3$

Here are the transformations in order:

1. Vertical Reflection $y = x^2 \rightarrow y = -x^2$
2. Horizontal shift of 1 unit left $\rightarrow y = -(x + 1)^2$
3. Vertical shift of 3 units down $\rightarrow y = -(x + 1)^2 - 3$

e) $y = 2^{3x+1} \rightarrow x = 2^{3y+1}$

This is a "INVERSE REFLECTION", will be taught in section 1.6. IN general, with Inverse reflections, the "x" and "y" coordinates will switch, meaning "x" coordinates will become "y" coordinates and then "y" coordinates will become "x" coordinates. (a,b) → (b,a).

f) $y = \frac{1}{x} \rightarrow y = \frac{-1}{-x + 5}$

There are several ways to recognize the transformations here:

1. Easiest method: Factor and then simplify first!!! $y = \frac{1}{x} \rightarrow y = \frac{-1}{-(x - 5)} = \frac{1}{x - 5}$. The negative signs will cancel out and then it's only a horizontal shift of 5 units right!.....
2. Standard method: Recognize all the negative signs as a reflection.....

   i) Vertical reflection: $y = \frac{1}{x} \rightarrow y = \frac{-1}{x}$

ii) Horizontal shift of 5 left!!... (yes left) $\rightarrow y = \dfrac{-1}{x+5}$

iii) Horizontal reflection.... $\rightarrow y = \dfrac{-1}{-x+5}$

7. Given the following transformation, $y = f(x) \rightarrow y = f(-x)$, which equation below will remain the same?

i) $y = x^2$ ii) $y = x^3 + 2x^2$ iii) $y = \sqrt{x^2}$ iv) $y = \dfrac{1}{2x+3}$ vi) $y = \left|3\left(2^x\right)\right|$

The question is asking which of the functions will look exactly the same when it goes through a horizontal reflection. This usually happens when the graph is symmetrical about the Y-axis. Algebraically, this happens when "x" is squared, $x^2 = (-x)^2$

8. Given each equation for $y = f(x)$, indicate the new equation after the transformations in the order stated:

For these types of questions, take the time to write out the transformations in steps. Ie: How is the y or "x" variable changed in each step. Then apply these changes in your equation sequentially.

a) $f(x) = 2x+3$ $S1:\quad y = 2(-x)+3$ $\quad y = -2x+3$ $S2:\quad y = -2(x-3)+3$ $S3:\ y-2 = -2(x-3)+3$ $\quad y = -2(x-3)+5$	1. A horizontal reflection over the Y-axis $x \longrightarrow -x$ 2. A shift of 3 units right $x \longrightarrow x-3$ 3. A shift of 2 units up $y \rightarrow y-2$
b) $f(x) = \dfrac{2}{3}(x-1)^2+1$ $S1:\quad -y = \dfrac{2}{3}(x-1)^2+1$ $\quad y = -\dfrac{2}{3}(x-1)^2-1$ $S2:\quad y = -\dfrac{2}{3}((x+2)-1)^2-1$ $\quad y = -\dfrac{2}{3}(x+1)^2-1$ $S3:\ y+6 = -\dfrac{2}{3}(x+1)^2-1$ $\quad y = -\dfrac{2}{3}(x+1)^2-7$	1. A vertical reflection over the X-axis $y \longrightarrow -y$ 2. A shift of 2 units left $x \longrightarrow x+2$ 3. A shift of 6 units down $y \longrightarrow y+6$

c) $f(x)=\sqrt{x+2}-3$ $y=\sqrt{x+2}-3$ $S1: x=\sqrt{y+2}-3$ $S2: x-4=\sqrt{y+2}-3$ $x=\sqrt{y+2}+1$ $S3: x=\sqrt{(y-6)+2}+1$ $x=\sqrt{y-4}+1$	1. A reflection over the Y=x line "x" and "y" variables are switched 2. A shift of 4 units left x → x - 4 3. A shift of 6 units up y → y – 6
d) $f(x)=5^x-1$ $y=5^x-1$ $S1: \quad -y=5^{-x}-1$ $y=-5^{-x}+1$ $S2: \quad y=-5^{-(x-3)}+1$ $y=-5^{-x+3}+1$ $S3: y+11=-5^{-x+3}+1$ $y=-5^{-x+3}-10$	1. A reflection in both the "x" and "y" axis X → -x and y → -y 2.A shift of 3 units right X → x – 3 3. A shift of 11 units down y → y + 11
e) $x^2+y^2=9$	1. A shift of 3 units right 2. A shift of 2 units up 3. A reflection over the "y" axis,
f) $y=\frac{1}{x+2}-3$	1. A shift of 2 units left, 2. A shift of 6 units down 3. A reflection in the line $y=x$,
g) $y=x^4+x^3-2x+1$	1. A reflection in the line $y=x$ 2. A shift of 6 units down
h) $y=\left\|\frac{1}{x-1}\right\|+3$	1. A reflection in the "y" axis 2. A shift of 4 units right 3.A shift of 11 units up 4. A reflection over the x-axis.

i) $y = x^3 - 3x$	1. A horizontal reflection over the Y-axis 2. Then an inverse reflection over the line $y = x$

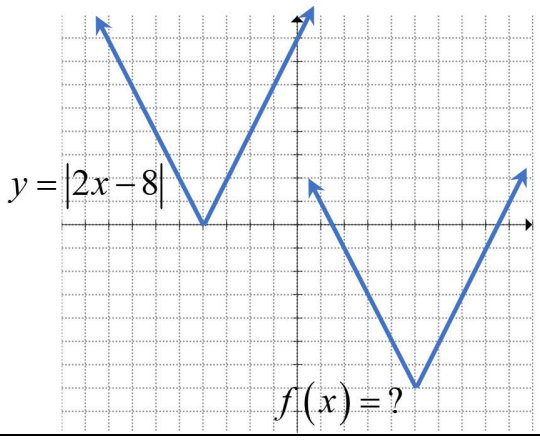
9. Given that the coordinates (a,b) are on the function $y = f(x)$. Indicate what this coordinate will become when y=f(x) is transformed to each of the functions below:
For these types of questions, remember that "a" and "b" are just numbers, and they represent the "x" coordinate and "y" coordinate respectively. First, indicate the transformations required to go from y=f(x) to each of the functions below. 2nd write down how the coordinates will be affected. 3rd: Apply these operations to the coordinates.

a) $y = f(-x)$ There's only one transformation: x → -x This is a horizontal reflection. So the "x" coordinate will be multiplied by -1. (a,b) → (-a, b)	b) $y = -f(x)$ This is a vertical reflection. Y → -y The y coordinates will be multiplied by -1 (a,b) → (a, -b)
c) $y = -f(x) + 2$ When reading the transformations, one way to keep it simple is to go from left to right. 1. Vertical reflection : "y" coordinates multiply by -1 2. Vertical shift of 2 units up "y" coordinates increase by 2 (a,b) → (a, -b) → (a, -b + 2)	d) $y = -f(-x) + 2$ 1. Vertical reflection "y" coordinates multiply by -1 2. Horizontal reflection "x" coordinates multiply by -1 3. Vertical shift of 2 up "y" coordinates increase by 2 (a,b) → (a,-b) → (-a,-b) → (-a, -b + 2)
e) $y = -f(x+4) - 7$ 1. Vert. Refl. (a, –b) 2. Hor. Shift 4 left (a – 4, – b) 3. Ver. Shift of 7 down (a – b, -b – 7)	f) $y = f(-(x-5))$ 1. Hor. Refl. (–a, b) 2. Hor Shift 5 right ((–a + 5, b)
g) $y = -f(-x+7)$ This one has two different ways. The standard method is to factor the negative sign inside the brackets first. $y = -f(-(x-7))$. Ask me this question in class if you are interested in learning a different way.	h) $y = f(-(x+9))$

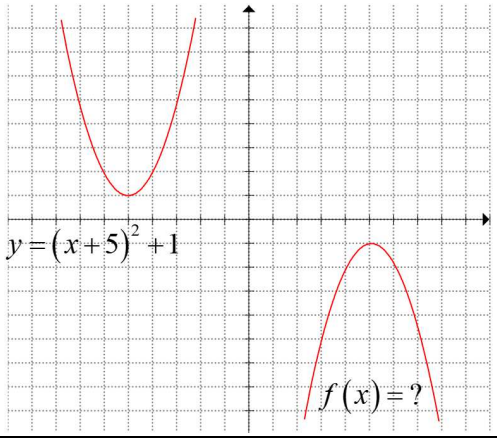
i) $-y = f(-x+3)$	j) $11+y = f(-x+1)+2$

10. Given the graph of $y = f(x)$ and the graph after transformation, what is the equation of the new graph?

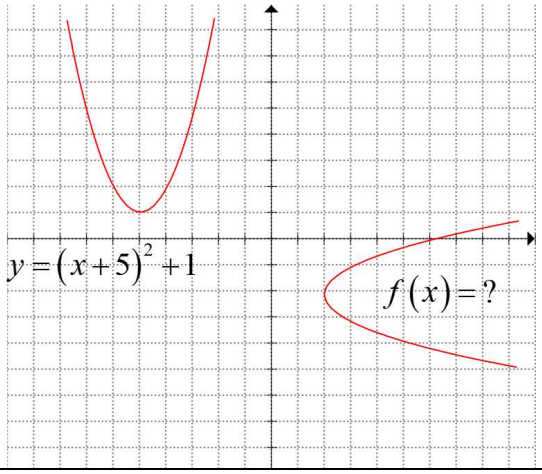
a)



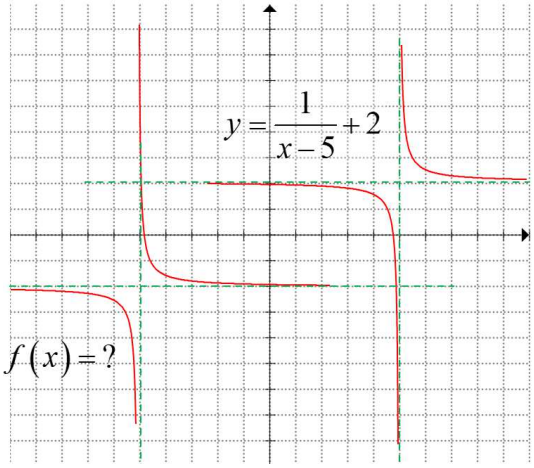
b.



c.



d.



11. Given that $f(x) = 2^x$ and $f_2(x) = 0.5^x$. What are all the transformation required for $f(x)$ to become $f_2(x)$?

12. Given that $f(x)=\sqrt{x}$ and $f_2(x)=-\sqrt{-x+3}+4$. What are all the transformation required for $f(x)$ to become $f_2(x)$? List them in order.

13. If the function $f(x)=x^2+8x+16$ is shifted 4 units up, 3 right, and reflected over the x-axis, the equation is now: $f(x)=a(x+b)^2+c$, what is the value of $a+b+c$?